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The Solution Space of the Unitary Matrix Model String Equation and the Sato Grassmannian

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Abstract. The space of all solutions to the string equation of the symmetric unitary one-matrix model is determined. It is shown that the string equation is equivalent to simple conditions on points V_1 and V_2 in the big cell $\mathrm{Gr}^{(0)}$ of the Sato Grassmannian Gr. This is a consequence of a well-defined continuum limit in which the string equation has the simple form $[\mathscr{P}, \mathscr{Q}_-] = 1$, with $\mathscr{P}$ and $\mathscr{Q}_-$ 2×2 matrices of differential operators. These conditions on V_1 and V_2 yield a simple system of first order differential equations whose analysis determines the space of all solutions to the string equation. This geometric formulation leads directly to the Virasoro constraints L_n $(n \geqq 0)$, where L_n annihilate the two modified-KdV τ-functions whose product gives the partition function of the Unitary Matrix Model.

1. Introduction

Matrix models form a rich class of quantum statistical mechanical systems defined by partition functions of the form $\int dM\, e^{-\frac{N}{\lambda}\mathrm{tr}\, V(M)}$, where M is an $N \times N$ matrix and the Hamiltonian $\mathrm{tr}\, V(M)$ is some well defined function of M. They were originally introduced to study complicated systems, such as heavy nuclei, in which the quantum mechanical Hamiltonian had to be considered random within some universality class [1, 4].

Unitary Matrix Models (UMM), in which M is a unitary matrix U, form a particularly rich class of matrix models. When $V(U)$ is self adjoint we will call the model symmetric. The simplest case is given by $V(U) = U + U^\dagger$ and describes two dimensional quantum chromodynamics [5–7] with gauge group $U(N)$. The partition function of this theory can be evaluated in the large-N (planar) limit in which N is taken to infinity with $\lambda = g^2 N$ held fixed, where g is the gauge coupling. The theory has a third order phase transition at $\lambda_c = 2$ [6]. Below λ_c the eigenvalues $e^{i\alpha_j}$ of U lie within a finite domain about $\alpha = 0$ of the form $[-\alpha_c, \alpha_c]$ with $\alpha_c < \pi$. The

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