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Irreducible Highest Weight Representations of Quantum Groups $U_q(gl(n, C))$

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Abstract. Explicit recurrence formulas of canonical realization (boson representation) for quantum enveloping algebras $U_q(gl(n, C))$ are given. Using them, irreducible highest weight representations of $U_q(gl(n, C))$ are obtained as restriction of representation on Fock space to invariant subspace generated by vacuum as a cyclic vector.

1. Introduction

The question of irreducible representations of quantum enveloping algebras was recently treated in a number of papers [1]. For irreducible highest weight representations (h.-w.irreps) it is known that their properties do not substantially differ (at least if q is not a root of unity) from the usual Lie algebra case. Especially it is proved [2] that a h.-w.irrep of the quantum enveloping algebra is uniquely determined (up to isomorphism) by its highest weight. It is also known for which highest weights these representations are finite dimensional.

These results were obtained by generalization of methods from the theory of highest weight representations of semisimple Lie algebras by means of which the construction of the explicit form of the highest weight representation for quantum enveloping algebras is, in principle, possible too.

In this paper we perform, in fact, this construction for the quantum enveloping algebra $U_q(gl(n, C)) \supset U_q(sl(n, C))$ defined in [3]. We do not, however, describe the details of construction (for the case of simple Lie algebras see [4]) but present the final formulas for direct verification.

In these formulas the generators of the algebra $U_q(gl(n+1, C))$ are expressed by means of n canonical boson pairs, one complex parameter and auxiliary representation of the algebra $U_q(gl(n, C))$. This recurrence character of formulas (1) is, by our opinion, its first interesting feature. It makes it possible, e.g. to obtain for special weights a simpler form of representation in comparison with general cases (see Concluding Remarks). The second advantage is that the invariant subspace with vacuum as cyclic vector is an irreducible one.