Commun. Math. Phys. 148, 403–416 (1992)

Communications in
Mathematical
Physics

Vertex Operator Representation of Some Quantum Tori Lie Algebras

M. Golenishcheva-Kutuzova [1] **and D. Lebedev** [2,*]

[1] Moscow Institute of Radiotechnology, Electronics and Automatics, Moscow, Russia
[2] Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy

Received December 2, 1991

Abstract. We are defining the trigonometric Lie subalgebras in $\bar{X}_\infty = \bar{A}_\infty(\bar{B}_\infty, \bar{C}_\infty, \bar{D}_\infty)$ which are the natural generalization of the well known Sin-Lie algebra. The embedding formulas into $\bar{X}_\infty$ are introduced. These algebras can be considered as some Lie algebras of quantum tori. An irreducible representation of A, B series of trigonometric Lie algebras is constructed. Special cases of the trigonometric Lie factor algebras, which can be considered as a quantum (preserving Lie algebra structure) deformation of the Kac-Moody algebras are considered.

1. Introduction

The trigonometric Sin-Lie algebra [1] is the one-dimensional extension of the quantum (Weyl-Moyal) [2]) deformation of the Poisson Lie algebra on the two-torus [3]. In the explicit realization [1] it is defined by the generators $T_{\bar{n}}$, the central element c and relations

$$[T_{\bar{n}}, T_{\bar{m}}] = 2i \sin \hbar_1 (\bar{n} \times \bar{m}) T_{\bar{n}+\bar{m}} + n_1 \delta_{\bar{n}+\bar{m}, \bar{0}} c, \tag{1}$$

where $\bar{n}$ and $\bar{m}$ are vectors belonging to a square integer lattice $\mathbf{Z}^2 \backslash (0,0)$, $\bar{n} \times \bar{m} = n_1 m_2 - m_1 n_2$ and $\hbar_1$ is an arbitrary real parameter.

The Lie algebra (1) is associated with an associative C^*-algebra, usually called irrational rotation algebra $A_{\hbar_1}$, which defines the noncommutative two-torus [4]. More precisely the C^*-algebra $A_{\hbar_1}$ is generated by two unitary operators U_1 and U_2 and the relation

$$U_2 U_1 = q^2 U_1 U_2 , \quad q = e^{i\hbar_1} .$$

* Permanent address: Institute of Theoretical and Experimental Physics 117259, Moscow, Russia