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Degeneracy in Loop Variables

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Abstract. The small algebra of loop functionals, defined by Rovelli and Smolin, on the Ashtekar phase space of general relativity is studied. Regarded as coordinates on the phase space, the loop functionals become degenerate at certain points. All the degenerate points are found and the corresponding degeneracy is discussed. The intersection of the set of degenerate points with the real slice of the constraint surface is shown to correspond precisely the Goldberg-Kerr solutions. The evolution of the holonomy group of Ashtekar's connection is examined, and the complexification of the holonomy group is shown to be preserved under it. Thus, an observable of the gravitational field is constructed.

1. Introduction

With the introduction by Ashtekar [1–4] of an $SL(2,\mathbb{C})$ connection $A_a{}^A{}_B$ and a densitized triad $\sigma^{aA}{}_B$ as new variables in canonical general relativity, the phase space takes on the appearance of a Yang-Mills gauge theory. A connection as a configuration space variable suggests the using of the parallel transport as the main device to construct gauge invariant quantities. The parallel transport of an arbitrary vector in the spinor space around all closed loops in the base manifold defines the holonomy group of the Yang-Mills connection. Elements of the holonomy group are given by the path ordered exponential of the integral of the connection A_a around closed loops. The traces of the holonomy integrals are gauge invariant functionals on the phase space, the Wilson loops. These functionals, identified as T^0, and the traces of the product of the holonomy integrals and the σ^a, T^1 functionals, form a closed Poisson bracket algebra. Therefore, they may be considered to be new configuration and momentum variables, thus new coordinates on the phase space. In fact, the use of these variables as new phase space coordinates was first introduced by Jacobson and Smolin [5] for the Ashtekar phase space for general relativity. Rovelli and Smolin [6] then showed that functionals T^n defined by the traces of the product of the holonomy integrals on polynomials homogeneous of degree n in the σ^a form a graded algebra.