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Selection Rules for Topology Change*

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Abstract. It is shown that there are restrictions on the possible changes of topology of space sections of the universe if this topology change takes place in a compact region which has a Lorentzian metric and spinor structure. In particular, it is impossible to create a single wormhole or attach a single handle to a spacetime but it is kinematically possible to create such wormholes in pairs. Another way of saying this is that there is a $\mathbb{Z}_2$ invariant for a closed oriented 3-manifold Σ which determines whether Σ can be the spacelike boundary of a compact manifold M which admits a Lorentzian metric and a spinor structure. We evaluate this invariant in terms of the homology groups of Σ and find that it is the mod2 Kervaire semi-characteristic.

Introduction

There has been great interest recently in the possibility that the topology of space may change in a semi-classical theory of quantum gravity in which one assumes the existence of an everywhere non-singular Lorentzian metric $g^L_{\alpha\beta}$ of signature $-+++$. In particular, Thorne, Frolov, Novikov and others have speculated that an advanced civilization might at some time in our future be able to change the topology of space sections of the universe so that they developed a wormhole or handle [1–3]. If one were to be able to control such a topology change, it would have to occur in a compact region of spacetime without singularities at which the equations broke down and without extra unpredictable information entering the spacetime from infinity. Thus if we assume, for convenience, that space is compact now, then the suggestion amounts to saying that the 4-dimensional spacetime manifold M, which we assume to be smooth and connected, is compact with boundary $\partial M = \Sigma$ consisting of 2 connected components, one of which has topology S^3 and the other of which has topology $S^1 \times S^2$, and both are spacelike

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