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Characters and Fusion Rules for W-Algebras via Quantized Drinfeld–Sokolov Reduction

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Abstract. Using the cohomological approach to W-algebras, we calculate characters and fusion coefficients for their representations obtained from modular invariant representations of affine algebras by the quantized Drinfeld–Sokolov reduction.

0. Introduction

The study of extended conformal algebras has been playing an increasingly important role in the recent development of conformal field theory. Among them the W-algebras have attracted much attention in the past few years. The first example of a W-algebra was discovered by Zamolodchikov [37] in an attempt to classify extended conformal algebras with two generating fields. (Further classification of W-algebras generated by two or three fields may be found in [8, 9].) There have been developed several approaches since then to the construction of a general W-algebra.

In the series of papers [15–17, 31] Fateev, Zamolodchikov and Lukyanov defined W-algebras associated to simple finite-dimensional Lie algebras $\bar{\mathfrak{g}}$ of type A_ℓ and D_ℓ by explicitly quantizing the corresponding Miura transformations and derived some results on their "minimal" representations. They put results in a form suitable for an arbitrary simply laced $\bar{\mathfrak{g}}$. At the same time Bilal and Gervais studied W-algebras as the algebras of symmetries of Toda theories [7].

In [2, 9, 11] the W-algebras appeared as the chiral algebras in coset models. In [34, 1] they also appeared in an attempt to generalize the Sugawara construction to higher degree Casimirs. All these constructions are closely related to the invariants of the Weyl group $\bar{W}$ of $\bar{\mathfrak{g}}$, hence the name W-algebra.

We adopt the point of view of the paper [21] by Feigin and one of the authors of the present paper, where the W-algebra $W(\bar{\mathfrak{g}})$, associated to any simple

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