On the *K*-Property
of Some Planar Hyperbolic Billiards

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Abstract. The *K*-property is demonstrated for a class of planar billiards satisfying Wojtkowski's principles. Their boundary may consist of convex-scattering, concave and linear pieces. Earlier Wojtkowski showed that these billiards had non-zero Lyapunov exponents.

1. Introduction

A highly intriguing and actual aim of the theory of Hamiltonian dynamical system is to understand the nature of coexistence of integrable and nonintegrable behaviors. There is a natural concensus that the simplest and most hopeful case to be studied is the two-dimensional one but there are different views as to whether which models are easier to attack. Two most concurrent families are billiards and standard maps. [In fact, an interesting example was constructed in Przytycki (1982).]

Billiards show a rich variety of phase portraits also encountered in general Hamiltonian dynamical systems. Nevertheless, their simpler geometrical properties might help to understand this variety more easily and serve as a starting point to learn more about general systems.

In 1979 Bunimovich proved the ergodicity of a billiard in a stadium. After that Wojtkowski (1986) constructed an extension of the class of billiards considered by Bunimovich. Later Markarian (1988) gave another extension of this class of billiards. Both Wojtkowski and Markarian proved that billiards in their classes had non-zero Lyapunov exponents. In the present note we demonstrate that billiards satisfying Wojtkowski's principles are, in fact, ergodic and even *K*-systems. A better understanding of the mechanism of ergodicity is hoped to provide tractable models where the nature of coexistence of integrable and non-integrable domains in the phase space can be revealed.

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