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Ergodic Properties of Plane Billiards with Symmetric Potentials

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Abstract. We study chaotic behaviour of the motion of a particle moving like in a billiard table outside some disks where a symmetric potential acts. Quadratic forms introduced in (Markarian, 1988) to study non-vanishing Lyapunov exponents are used.

1. Introduction

A. The purpose of this paper is to prove some ergodic properties (see C. below) of dynamical systems defined by the motion of a point mass in a bounded connected region Q of the plane. The particle moves like in a billiard table outside some disk contained in Q, and under the action of symmetric potentials inside them.

B. This problem was discussed (on the 2-torus) in Sinai (1963). In Kubo (1976) and Kubo and Murata (1981) it is proved that some conditions on the rotation function (see Sect. 2 of this paper) are sufficient for the ergodicity (and the Bernoulli property) of the dynamical system on the torus; Kubo mentions some repelling potentials that verify these conditions.

During the International Conference and Workshop on Dynamical Systems, I.M.P.A., Rio de Janeiro, August 1989, Victor Donnay announced some results (obtained by himself and Carlangelo Liverani) on the same subject, using invariant cone techniques; and Yacob Sinai suggested me to study related problems using quadratic forms, a method I used to deduce conditions on the boundaries for chaotic billiards.

C. Let μ be a probability measure on a compact 2-manifold M, K a subset such that $\mu(K) = 0$, $H = M\backslash K$ and $T: H \to H$ the restriction to H of a $\mathbb{C}^r$-diffeomorphism, $r \geq 1$, defined on an open subset of M that preserved the measure μ. We will assume that

$$\log^+ \|(T^{\pm 1})'_x\| \in L^1(H, \mu) \qquad (\log^+ s = \max\{\log s, 0\}),$$

a condition that permits to apply the ergodic multiplicative theorem of Oseledets.

$\Sigma(T)$ will denote the Pesin region, that is the set of regular points that have only non-zero Lyapunov exponents.