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Fluctuations of One Dimensional Ginzburg–Landau Models in Nonequilibrium

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Abstract. We study the fluctuation of one dimensional Ginzburg–Landau models in nonequilibrium along the hydrodynamic (diffusion) limit. The hydrodynamic limit has been proved to be a nonlinear diffusion equation by Fritz, Guo–Papanicolaou–Varadhan, etc. We proved that if the potential is uniformly convex then the fluctuation process is governed by an Ornstein–Uhlenbeck process whose drift term is obtained by formally linearizing the hydrodynamic equation.

I. Introduction

In this paper we study the nonequilibrium fluctuations for a system of charges or spins in a one dimensional periodic lattice. Our model is generally known as a time dependent Ginzburg–Landau model (TDGL) with a conservation law. The dynamics in this model are governed by a conservation law together with random noise which also conserve the total charge. The resulting process is gradient and reversible with respect to a family of time *independent* Ginzburg–Landau (Gibbs) measures (TIGL).

The large scale behavior of TDGL model has been a subject of much interest. In particular, one can study the limit of the charge density under the diffusive type scalings – the hydrodynamical scaling limit of the charge density. Under various assumptions and generalities it has been proved [4, 5, 6, 8] that the hydrodynamic limit of the charge density in TDGL is governed by a nonlinear diffusion equation – the macroscopic equation. The macroscopic equation has also been studied considerably before and properties such as uniqueness, existence and a priori bounds are well known [see 8 and references therein]. Thus, the hydrodynamic

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