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A Model System for Strong Interaction Between Internal Solitary Waves

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Abstract. A mathematical theory is mounted for a complex system of equations derived by Gear and Grimshaw that models the strong interaction of two-dimensional, long, internal gravity waves propagating on neighboring pycnoclines in a stratified fluid. For the model in question, the Cauchy problem is of interest, and is shown to be globally well-posed in suitably strong function spaces. Our results make use of Kato's theory for abstract evolution equations together with somewhat delicate estimates obtained using techniques from harmonic analysis. In weak function classes, a local existence theory is developed. The system is shown to be susceptible to the dispersive blow-up phenomenon investigated recently by Bona and Saut for Korteweg-de Vries-type equations.

1. Introduction

This paper is concerned with the initial-value problem

$$\begin{cases} u_t + uu_x + u_{xxx} + a_3 v_{xxx} + a_1 vv_x + a_2 (uv)_x = 0, \\ b_1 v_t + rv_x + vv_x + v_{xxx} + b_2 a_3 u_{xxx} + b_2 a_2 uu_x + b_2 a_1 (uv)_x = 0, \\ u(x,0) = u_0(x), \\ v(x,0) = v_0(x), \end{cases} \tag{1.1}$$

where a_1, a_2, a_3, b_1, b_2 and r are real constants with b_1, b_2 positive, $u = u(x,t)$, $v = v(x,t)$ are real-valued functions of the two real variables x and t, and subscripts