Commun. Math. Phys. 143, 43–83 (1991)

Communications in Mathematical Physics

Quasiperiodic Motions in Superquadratic Time-Periodic Potentials

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Abstract. It is shown that for a large class of potentials on the line with superquadratic growth at infinity and with the additional time-periodic dependence all possible motions under the influence of such potentials are bounded for all time and that most (in a precise sense) motions are in fact quasiperiodic. The class of potentials includes, as very particular examples, the exponential, polynomial and much more. This extends earlier results and gives an answer to a problem posed by Littlewood in the mid 1960's. Along the way machinery is developed for estimating the action-angle transformation directly in terms of the potential and also some apparently new identities involving singular integrals are derived.

1. Introduction and the Results

In the early 1960's Littlewood [LI] asked whether or not the solutions of the Duffing-type equations

$$\ddot{x} + g(x) = p(t), \quad \text{where} \quad p(t+1) = p(t) \tag{1.0}$$

are bounded for all time, i.e. whether there are resonances that might cause the amplitude of the oscillations to increase without bound. In this paper we study the more general problem of describing the behavior of solutions of the system of the form

$$\ddot{x} + V_x(x, t) = 0, \tag{1.1}$$

which is a Hamiltonian system governing the motion of a particle on the line subject to a time-periodic force. We show that under appropriate growth assumptions on V for large x, the system is near-integrable for large amplitudes in the sense that "most" large amplitude solutions are quasiperiodic and all solutions are bounded for all time with no smallness assumptions on the time-dependence of $V(x, t)$. Intuitively, one might expect that if $V(x, t)$ is superquadratic in x, then the larger amplitude solutions oscillate faster giving rise to a twist in the Poincaré map of the $(x, \dot{x})$ plane and thus a hope of applying Moser's twist theorem to