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Morita Equivalence of Poisson Manifolds

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Abstract. Poisson manifolds are the classical analogue of associative algebras. For Poisson manifolds, symplectic realizations play a similar role as representations do for associative algebras. In this paper, the notion of Morita equivalence of Poisson manifolds, a classical analogue of Morita equivalence of algebras, is introduced and studied. It is proved that Morita equivalent Poisson manifolds have equivalent "categories" of complete symplectic realizations. For certain types of Poisson manifolds, the geometric invariants of Morita equivalence are also investigated.

Introduction

Poisson manifolds are the classical analogue of C^*-algebras (or noncommutative algebras). One can find counterparts in Poisson geometry for many concepts in C^*-algebras. For instance, representations of C^*-algebras correspond to symplectic realizations of Poisson manifolds, traces of C^*-algebras to invariant measures on Poisson manifolds, automorphism groups of C^*-algebras to Poisson flows on Poisson manifolds, etc. The similarities between these two distinct subjects are more than conceptual; they are also reflected in the techniques and methods of studying these subjects. Therefore, it would be useful in the study of Poisson geometry to develop comparable techniques to those used in the theory of C^*-algebras, such as the theory of Morita equivalence.

The theory of Morita equivalence goes back to Morita [Mo] in the 1950's, who proved the fundamental theorem: two rings have equivalent categories of left modules if and only if there exists an equivalence bimodule for the rings. This concept of equivalence was first generalized to the context of C^*-algebras, in the name of strong Morita equivalence, by Marc Rieffel [Rie1–Rie4]. It turns out now to be one of the most important equivalence relations in C^*-algebras, playing a crucial role in understanding the structures of some C^*-algebras, such as

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