

# Asymptotics of a $\tau$ -Function Arising in the Two-Dimensional Ising Model<sup>★</sup>

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**Abstract.** The short-distance asymptotics of the  $\tau$ -function associated to the 2-point function of the two-dimensional Ising model is computed as a function of the integration constant defined from the long-distance behavior of the  $\tau$ -function. The result is expressible in terms of the Barnes double gamma function (equivalently, the Barnes  $G$ -function).

## 1. Introduction and Summary of Results

If  $\xi = \xi(T)$  is the correlation length,  $T$  is temperature, and  $\langle \sigma_{00} \sigma_{MN} \rangle$  is the spin-spin correlation function for the two-dimensional Ising model,<sup>1</sup> then in the scaling limit, defined by

$$\xi \rightarrow \infty, \quad R = (M^2 + N^2)^{1/2} \rightarrow \infty,$$

such that

$$t = \frac{R}{\xi} \quad \text{is fixed,}$$

it is known [3, 6] (see also [4]) that

$$\lim R^{1/4} \langle \sigma_{00} \sigma_{MN} \rangle = F_{\pm}(t), \quad (1.1)$$

where  $+$ ( $-$ ) denotes the limit is taken above (below) the critical temperature  $T_c$ . Furthermore, the scaling functions  $F_{\pm}(t)$  are given by [3, 6]

$$F_{\pm}(t) = 2^{3/8} t^{1/4} \tau_{\pm} \left( t, \frac{1}{\pi} \right), \quad (1.2)$$

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<sup>1</sup> For simplicity of presentation we assume that the horizontal and vertical interactions are equal