Commun. Math. Phys. 141, 353–380 (1991)

Communications in
Mathematical
Physics

Continuity Properties of the Electronic Spectrum of 1D Quasicrystals

J. Bellissard*, B. Iochum*, and D. Testard**

Centre de Physique Théorique***, C.N.R.S. Luminy Case 907, F-13288 Marseille Cedex 9, France

Received November 30, 1990; in revised form March 25, 1991

Abstract. In this paper we consider operators $H(\alpha, x)$ defined on $l^2(\mathbb{Z})$ by

$$H(\alpha, x)\psi(n) = \sum_{m \in \mathbb{Z}} t_m \circ \phi^{-n}(\alpha, x)\psi(n-m),$$

where $\phi(\alpha, x) = (\alpha, x-\alpha)$, t_m is in the algebra of bounded periodic functions on $\mathbb{R}^2$ generated by the characteristic functions of the sets

$$\phi^n\{(\alpha, x) \in \mathbb{R}^2 \mid 1-\alpha \leqq x < \alpha \pmod 1\}.$$

This class of hamiltonian includes the Kohmoto model numerically computed by Ostlund and Kim, where the potential is given by

$$v_{\alpha,x}(n) = \lambda\chi_{[1-\alpha, 1[}(x+n\alpha), \qquad n \in \mathbb{Z},\ x,\ \lambda,\ \alpha \in \mathbb{R}$$

(see [B.I.S.T.]). We prove that the spectrum (as a set) of $H(\alpha, x)$ varies continuously with respect to α near each irrational, for any x. We also show that the various strong limits obtained as α converges to a rational number $\frac{p}{q}$ describe either a periodic medium or a periodic medium with a localized impurity. The corresponding spectrum has eigenvalues in the gaps and the right and left limits as $\alpha \to \frac{p}{q}$ do not coincide, for the Kohmoto model. The results are obtained through C^*-algebra techniques.

1. Introduction

Let us consider the following discrete one dimensional Schrödinger hamiltonian with quasiperiodic potential, acting on $l^2(\mathbb{Z})$ and given by

$$H(\alpha, x, \lambda)\psi(n) = \psi(n+1) + \psi(n-1) + \lambda v_{\alpha,x}(n)\psi(n), \tag{1}$$

* And Université de Provence, Marseille
** And Université d'Aix-Marseille II, Luminy
*** Laboratoire propre L.P. 7061 du Centre National de la Recherche Scientifique