

Nonlocal Integrable Partners to Generalized MKdV and Two-Dimensional Toda Lattice Equation in the Formalism of a Dressing Method with Quantized Spectral Parameter

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Abstract. Two new hierarchies, $MILW_2$ and a two-dimensional nonlocal Toda lattice are constructed. The characteristic property of the first one is the connection with the ILW_2 hierarchy by means of $gl(2)$ Miura transformation. On the other hand, $MILW_2$ equations turn out to be symmetry equations for a two-dimensional nonlocal Toda lattice. A new version of the dressing technique with quantized spectral parameter is proposed.

1. Introduction

This paper is a direct continuation of our previous paper [1]. In [1] for the second representative ($n=2$) of the Intermediate Long Waves (ILW_n) hierarchy [2], the zero-curvature representation with a noncommutative spectral parameter $\hat{\lambda} = \lambda e^{-2i\hbar\partial_x}$ has been constructed. Here λ and \hbar are certain numerical parameters and $\partial_x = d/dx$. (The pioneering papers in the theory of the ILW equation are [3].) This has been achieved by means of a new version of the Zakharov-Shabat dressing technique with a noncommutative spectral parameter. Formally, the spectral parameter $\hat{\lambda}$ may be considered as a quantized spectral parameter λ , so that, in what follows we will refer to $\hat{\lambda}$ as a quantum spectral parameter.

It has also been explained in [1] that the ILW_n hierarchy appears as a special reduction of the KP hierarchy, which generalizes the well known reduction to KdV-type equation. Therefore, this hierarchy of equations lies, roughly speaking, at the same level with KdV type equations. From the other side, one can imagine the ILW_n hierarchy as an one-parameter deformation of the KdV equation [3]. We will say in this case that ILW type equations are the nonlocal integrable partners to KdV type equations.