Commun. Math. Phys. 141, 9–39 (1991)

Communications in
Mathematical
Physics

The Number Process as Low Density Limit of Hamiltonian Models

L. Accardi and Y. G. Lu *

Centro Matematico V. Volterra and Dipartimento di Matematica, Università di Roma II, Italy

Received June 22, 1989; in revised form February 13, 1991

Abstract. We study a nonrelativistic quantum system coupled, via a quadratic interaction [cf. formula (1.10) below], to a free Boson gas in the Fock state. We prove that, in the low density limit (z^2 = fugacity$\to 0$), the matrix elements of the wave operator of the system at time t/z^2 in the collective coherent vectors converge to the matrix elements, in suitable coherent vectors of the quantum Brownian motion process, of a unitary Markovian cocycle satisfying a quantum stochastic differential equation driven by some pure number process (i.e. no quantum diffusion part and only the quantum analogue of the purely discontinuous, or jump, processes). This proves that the number (or quantum Poisson) processes, introduced by Hudson and Parthasarathy and studied by Frigerio and Maassen, arise effectively as conjectured by the latter two authors as low density limits of Hamiltonian models.

0. Introduction

The study of the weak coupling (van Hove) or low density limit of quantum Hamiltonian systems is framed within the wider program of understanding the origins of irreversible behaviours in quantum phenomena. In this study three stages of developments can be recognized: (i) a first one, in which the driving scales of magnitude are individuated and the irreversible equations are deduced on a phenomenological basis (van Hove scaling, Pauli master equation, Wigner-Weiskopf approximation); (ii) a second one, in which the various types of master equations are deduced from Hamiltonian models; (iii) a third one, in which, starting from the same types of models as in (ii), one tries to derive not only the master equation, but the full quantum Langevin equation.

The master equation is an ordinary differential equation, describing the **reduced evolution** of the system, obtained from the full Heisenberg evolution by taking the

* On leave of absence from Beijing Normal University