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Interfaces in the Potts Model I: Pirogov–Sinai Theory of the Fortuin–Kasteleyn Representation

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Abstract. We develop a new analysis of the order–disorder transition in ferromagnetic Potts models for large number q of spin states. We use the Pirogov–Sinaï theory which we adapt to the Fortuin–Kasteleyn representation of the models. This theory applies in a rather direct way in our approach and leads to a system of non-interacting contours with small activities. As a consequence, simpler and more natural techniques are found, allowing us to recover previous results on the bulk properties of the model (which then extend to non-integer values of q) and to deal with non-translation invariant boundary conditions. This will be applied in a second part of this work to study the behaviour of the interfaces at the transition point.

1. Introduction

When we consider three phases, a, b and c, in thermal equilibrium, the situation may arise in which a layer of the phase c is developed at the boundary between the a and b phases in order to lower the surface tension (i.e. the free energy). Then two interfaces appear: one between a and c, the other between c and b. The interaction between these two interfaces leads to a large variety of physical phenomena.

A theoretical example of this situation is provided by the ferromagnetic q-state Potts model in dimension $d \geqq 2$. This model exhibits q ordered phases at low temperature and one disordered phase at high temperature. When the order–disorder transition is first order, which is the case when q is large enough, all phases

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