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Formulas for the Derivative and Critical Points of Topological Entropy for Anosov and Geodesic Flows*

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Abstract. This paper represents part of a program to understand the behavior of topological entropy for Anosov and geodesic flows. In this paper, we have two goals. First we obtain some regularity results for C^1 perturbations. Second, and more importantly, we obtain explicit formulas for the derivative of topological entropy. These formulas allow us to characterize the critical points of topological entropy on the space of negatively curved metrics.

I. Formulation of Results

The topological entropy, h_{TOP}, measures the exponential growth rate of the number of orbit segments distinguishable with arbitrarily fine but finite precision. In general, it behaves irregularly with respect to perturbations. In the discrete time case, Misiurewicz [Mi1] constructed examples showing that $h_{\mathrm{TOP}}:\mathrm{Diff}^{\infty}(M^n)\to\mathbb{R}$ is not continuous for $n\geqq 4$, as well as examples showing that h_{TOP} is not upper-semicontinuous in the C^k topology for $k<\infty$ in every dimension $n\geqq 2$ [Mi2]. Here $\mathrm{Diff}^{\infty}(M^n)$ denotes the space of C^{∞} diffeomorphisms on a compact n-dimensional manifold equipped with the C^{∞} topology. Yomdin [Y] and Newhouse [N] proved that $h_{\mathrm{TOP}}:\mathrm{Diff}^{\infty}(M^n)\to\mathbb{R}$ is upper semicontinuous. For $n=2$, Katok [K3] proved lower semicontinuity. By combining these two results, one sees that $h_{\mathrm{TOP}}:\mathrm{Diff}^{\infty}(M^2)\to\mathbb{R}$ is continuous. This result also holds for C^{∞} flows on three dimensional manifolds.

The structural stability of Anosov diffeomorphisms [A] implies that h_{TOP} is locally constant. For Anosov flows, the structural stability [A] involves a time

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