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On the Determinant of Elliptic Differential and Finite Difference Operators in Vector Bundles over S^1

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Abstract. For an elliptic differential operator A over S^1, $A = \sum_{k=0}^{n} A_k(x)D^k$, with $A_k(x)$ in $\mathrm{END}(\mathbb{C}^r)$ and θ as a principal angle, the ζ-regularized determinant $\mathrm{Det}_\theta A$ is computed in terms of the monodromy map P_A, associated to A and some invariant expressed in terms of A_n and A_{n-1}. A similar formula holds for finite difference operators. A number of applications and implications are given. In particular we present a formula for the signature of A when A is self adjoint and show that the determinant of A is the limit of a sequence of computable expressions involving determinants of difference approximation of A.

1. Introduction and Summary of the Results

In this paper we study the determinant of elliptic differential operators on a complex vector bundle $E \xrightarrow{p} M$ of rank N over a compact oriented connected manifold M of dimension 1, as well as the determinants of its finite difference approximations. For this purpose we introduce a new invariant S_θ which, in the case of odd order self adjoint operators, calculates the η-invariant (Corollary 5.4).

In order to state the first main theorem we have to introduce the following notions for elliptic differential operators.

(1) The monodromy map P_A: Denote by $\Gamma(E)$ the smooth sections of $E \xrightarrow{p} M$. For an elliptic differential operator $A:\Gamma(E)\to\Gamma(E)$ of order $n \geqq 1$ consider the lift $\tilde{A}:\Gamma(\tilde{E})\to\Gamma(\tilde{E})$, where $\tilde{E} \xrightarrow{\tilde{p}} \tilde{M}$ is the pullback of $E \xrightarrow{p} M$ by the universal covering $\tilde{M}\to M$. Due to the ellipticity of A, the nullspace $\mathrm{Null}(\tilde{A})$ has dimension nN. The fundamental group $\pi_1(M,*)=\mathbb{Z}$ (with 1 corresponding to the orientation of M)

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