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Characterization of States of Infinite Boson Systems

I. On the Construction of States of Boson Systems

Karl-Heinz Fichtner and Wolfgang Freudenberg

Mathematische Fakultät, Friedrich-Schiller-Universität Jena, UHH 17.06, O-6900 Jena, Federal Republic of Germany

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Abstract. In a previous paper [11] it was shown that to each locally normal state of a boson system one can associate a point process that can be interpreted as the position distribution of the state. In the present paper the so-called conditional reduced density matrix of a normal or locally normal state is introduced. The whole state is determined completely by its position distribution and this function. There are given sufficient conditions on a point process Q and a function k ensuring the existence of a state such that Q is its position distribution and k its conditional reduced density matrix. Several examples will show that these conditions represent effective and useful criteria to construct locally normal states of boson systems. Especially, we will sketch an approach to equilibrium states of infinite boson systems. Further, we consider a class of operators on the Fock space representing certain combinations of position measurements and local measurements (observables related to bounded areas). The corresponding expectations can be expressed by the position distribution and the conditional reduced density matrix. This class serves as an important tool for the construction of states of (finite and infinite) boson systems. Especially, operators of second quantization, creation and annihilation operators are of this type. So, independently of the applications in the above context this class of operators may be of some interest.

1. Introduction

For a mathematical explanation of many structural effects, phase transitions, characterizations of equilibrium states, etc. of "large" quantum systems it turned out to be useful to have available mathematical models for infinite quantum systems. There are several approaches to the study of infinite particle systems. In quantum statistical mechanics a common approach is the concept of quasilocal C^*-algebras $\mathscr{A}$. Hereby $\mathscr{A}$ is the (norm completion of the) union of all local algebras representing measurements in bounded areas of the phase space. The local algebras are assumed to be (isomorphic to) the algebras of bounded linear