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On the Relation between Types of Local Algebras in Different Global Representations

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Abstract. Let $\{\mathscr{A}, \mathscr{A}(O), \mathbb{R}^d, \alpha\}$ be a theory of local observables. We show that there are relations between the Connes–von Neumann types of the algebras belonging to a different global representation. For example if one representation is the vacuum representation such that the wedge algebra is of type III_1 then this also is the case for other representations, provided these are connected with the vacuum by large translations.

1. Introduction

The type of local von Neumann algebras has been investigated in many papers (see e.g. [1, 6, 8, 9, 16]). These investigations are either based on the explicit knowledge of the algebra (free fields or generalized free fields), on the structure of lightlike translations, or on the assumption that the theory of local observables in question is connected to a Wightman field in some way or other. The latter group of results is obtained by assuming the existence of a scaling limit, which then gives rise to a free massless field whose local modular structure is known due to the analysis of Hislop and Longo [15] (for details see Fredenhagen [11], and Wollenberg [20]). In most of these investigations one finds that the Connes–von Neumann-type of the algebras under consideration is III_1.

We are interested in cases where the existence of scaling limits has not been proved, but where we have one of the following two situations: One is the case where we have an algebra and a subalgebra and in the other case we are considering two different representations. What we have in mind are cases where the two representations are connected by large translations, a situation more general than that appearing in the theory of superselection sectors initiated by Borchers [4] and developed by Doplicher, Haag, and Roberts [10] and by Buchholz and Fredenhagen [5].

We will assume that we are dealing with a C^*-algebra $\mathscr{A}$ invariant under a group α_s, $s \in \mathbb{R}$ and that we have two states, one ω_0 which is invariant, i.e.