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Indecomposable Modules Over the Virasoro Lie Algebra and a Conjecture of V. Kac*

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Abstract. We consider a class of indecomposable modules over the Virasoro Lie algebra that we call bounded admissible modules. We get results concerning the center and the dimensions of the weight spaces. We prove that these modules always contain a submodule with one-dimensional weight spaces. From this follows the proof of a conjecture of V. Kac concerning the classification of simple admissible modules.

Introduction

The Virasoro algebra $\mathscr{V}$ is the universal central extension of the complex Lie algebra W of polynomial vector fields on the circle [1]. W is also the Lie algebra of the group of diffeomorphisms of the circle. The Virasoro algebra plays a fundamental role in two dimensional conformal quantum field theory [2–5]. Therefore, the unitarizable $\mathscr{V}$-modules with highest or lowest weight have been extensively studied by many authors [5–8]. For the same reason it was of interest to determine whether the Verma modules are irreducible or not irreducible [9]. We can also notice that indecomposable Verma $\mathscr{V}$-modules occur in two dimensional conformal degenerate quantum field theory [4]. Besides highest or lowest weight $\mathscr{V}$-modules, another class of $\mathscr{V}$-modules was exhaustively classified by Kaplansky–Santharoubane [10]. These are the indecomposable $\mathscr{V}$-modules, where x_0 the rotation generator acts semisimply with one-dimensional eigenspaces. Such $\mathscr{V}$-modules have been previously introduced by Feigin–Fuchs [11]. The existence of the last type of $\mathscr{V}$-modules permits to confirm that the result of Kostrikin [12] concerning the simple Z-graduate Lie algebras of Cartan type other than W_1 (the algebra of the derivations of the polynomial ring $\mathbb{C}[X]$) is indeed false for W_1 as W_1 is a subalgebra of $\mathscr{V}$. Thus Victor Kac [13] conjectured the following theorem:

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