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The C^*-Algebra of Bosonic Strings*

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Abstract. We give a rigorous definition of Witten's C^*-string-algebra. To this end we present a new construction of C^*-algebras associated to special geometric situations (Kähler foliations) and generalize this later construction to the string case. Through this we get a natural geometrical interpretation of the occurrence of semi-infinite forms as well as the fermionic algebra structure. Using the (non-commutative) geometric concepts for investigating the string algebra we get a natural Fredholm module representation of dimension 26+.

1. Introduction

One of the nice features in the development of string theory turned out to be the simple ways in which strings can interact. For example the open bosonic strings can only either split or join. The joining of two strings to a new one may remind one of a product-like structure while the splitting may be compared to the factorization.

This idea was pushed forward by Witten in 1986, [Wi 1], where he introduced a groupoid structure for classical strings, see Sect. 1. In the Schrödinger picture of the first quantization of string theory one passes from the classical strings (continuous paths in $\mathbf{R}^{1,d-1}$) to wavefunctions on the space of classical strings. The groupoid structure yields in a canonical way a *-algebra-structure on the space of wavefunctions. The first part of this article gives a precision to Witten's definition and describes this (bosonic-) algebra explicitly.

Now the physics should not depend on the parametrization of the interacting strings. The reparametrization group is an infinite-dimensional Frechet-Lie-group, [Mi]. In the course of quantizing the theory we have to take care of the unphysical degrees of freedom caused by the symmetry. Usually this is done by introducing Faddeev-Popov-ghost-fields, which are fermionic fields, for dividng out the

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