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Strong Asymptotic Abelianness for Entropic K-Systems

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Abstract. We prove that in entropic K-systems of type II_1 the automorphism is strongly asymptotically abelian.

1. Introduction

In classical statistical mechanics, approach to equilibrium is a well-known consequence of the mixing properties of the physical systems. These properties manifest themselves in the clustering behaviour of correlation functions. The strongest version of clustering occurs in those systems called K-systems [1]. Since nature is governed by noncommutative laws we are led to try to formulate concepts analogous to that of abelian ergodic theory also in a quantum frame.

It is clear that finite quantum systems cannot exhibit any mixing, because of the spectral character of the Hamiltonian governing their evolution. But in the thermodynamic limit of an infinite system we can have representations of the algebra of operators such that the generator of the dynamics does not belong to the algebra. Then we are in a good position to find an approach to equilibrium without invoking a coupling to an external reservoir that should drive the system to equilibrium.

Asymptotically abelian systems with ergodic or even mixing properties, as defined in the literature, are characterized by certain typical decays of the two point correlation functions which resemble the weak or strong mixing typical of the abelian case. Recently the extension of the Kolmogorov–Sinai entropy to the quantum realm [2] has been used to concretely formulate a noncommutative version of K-systems [3] as systems with "complete memory loss." This definition reduces to the classical one for abelian systems and there provides a certain "strong" clustering property.

Also for quantum systems we shall prove that entropic K-systems that are of type II_1 guarantee strong asymptotic abelianness and therefore also clustering.

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