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# On the Initial Condition for Instanton Solutions

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**Abstract.** To each gauge equivalence class of both local and global framed (in the sense of Donaldson) self-dual solutions with the gauge group **U**(r) there is related the unique canonical initial condition (in the sense of Takasaki) and in this way the gauge freedom is eliminated. A geometric interpretation is given and consequently the complete transcription of the ADHM construction into the inverse scattering formalism is derived. As an application, an injection holomorphic mapping of the instanton moduli space into a finite-dimensional complex vector space is described and the loop group action on the transition functions is discussed. The results suggest the possibility of a new description of the framed instanton moduli spaces directly as algebraic sets.

## 1. Introduction

It is well known that the self-dual Yang-Mills equations admit two different approaches: one initiated by Ward [1] and based on the Penrose twistor transformation and the other, going back to Yang [2], Belavin and Zakharov [3], based on the inverse scattering method. The former one, insisting on global methods (cf. [4]), has succeeded in the ADHM construction [5]. The inverse scattering approach is essentially local. It insists on expressions in local coordinates and consequently it enables one to apply some more general methods such as the Bäcklund transformation [6, 7], the construction of an infinite algebra of symmetries due to Dolan [8] and Chau, Ge, Sinha and Wu [9] and the solution of the initial value problem due to Takasaki [10].

This paper addresses the initial condition in the sense of Takasaki and its geometric interpretation in the global case. The starting point is a more detailed discussion of the gauge transformations in the local formulation. The main point is to eliminate completely the gauge freedom. The geometric interpretation suggests a new way to describe the framed instanton moduli spaces (in the sense of Donaldson [11]) directly as algebraic sets. We recall that in the ADHM

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