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A Generalization of the Kac-Moody Algebras with a Parameter on an Algebraic Curve and Perturbations of Solitons

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Abstract. The Lie-algebraic approach for the dynamic systems associated with a generalization of the Kac-Moody algebras on Riemann surfaces is developed. A technique of solving the inverse scattering problem of operators with spectral parameters on Riemann surfaces is presented. Some equations associated with generalized Kac-Moody algebras are presented. The connection between their hamiltonian structure and deformed Lax representation is discussed as well as its applications to some special perturbations of integrable systems.

1. Introduction

The Lie-algebraic approach to the theory of completely integrable nonlinear equations makes it possible to connect a classical r-matrix formalism, representations of zero curvature and Poisson brackets of the coefficients of transition matrices. Kostant [1] was the first who proposed the method of construction of integrable systems by dividing a Lie algebra to a direct sum of two subalgebras $\mathfrak{G} = \mathfrak{G}_+ \oplus \mathfrak{G}_-$. Then in works of the Leningrad-school researchers (see [2] and reference in it) this method was adapted to a wide class of the Lie algebras including infinite dimensional ones. Almost all known nonlinear equations admitting the Lax representation were plunged into this approach.

In this paper the same approach is developed for some Lie algebras dividing as

$$\mathfrak{G} = \mathfrak{G}_+ \oplus \mathfrak{G}_0 \oplus \mathfrak{G}_-,$$

where $\mathfrak{G}_\pm$ are subalgebras. $\mathfrak{G}_0$ is a finite dimensional subspace. They are used in consideration of the operator bundles parametrized by point on an algebraic curve of genus $g > 0$. For a class of such operators the Gelfand-Lefitan-Marchenko approach to the solution of the inverse scattering problem was developed in [3]. In Sect. 3 its modified version will be given without proofs. As it follows from [4], the algebra of those operators inevitably includes $\mathfrak{G}_0$. It will be seen that most aspects of the Lie-algebraic approach require generalizations.