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Summability Method for a Horn-Shaped Region

Alexander Moroz

Institute of Physics ČSAV, Na Slovance 2, CS-180 40 Prague 8, Czechoslovakia

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Abstract. A formulation of the Nevanlinna-like theorem for a horn-shaped region is given. Product of functions obeying the hypotheses of the theorem is also shown to obey these hypotheses, i.e. the summability mechanism preserves nonlinear perturbative conditions such as unitarity of the Feynman series.

1. Introduction

There is a notoriously known fact that perturbation series in quantum theory are mostly divergent and can have at best the meaning of asymptotic series.

Unlike convergent power series the divergent asymptotic series do not determine a function uniquely. More precisely given an arbitrary sequence $\{a_n\}_0^\infty$ of complex numbers and an arbitrary sector-like domain D, there exists for some $\varepsilon>0$ a function $f(z)$, which is regular in $D_\varepsilon := D \cap \{z \mid |z|<\varepsilon\}$ and such that for every n,

$$\lim_{\substack{z\to 0\\ z\in D_\varepsilon}} (f(z)-a_0-\dots-a_n z^n)/z^{n+1}=a_{n+1}$$

exists, or equivalently

$$f(z)\sim \sum_{n=0}^{\infty} a_n z^n \qquad (z\to 0,\ z\in D_\varepsilon), \tag{1}$$

i.e. $\sum_{n=0}^{\infty} a_n z^n$ is an asymptotic series of $f(z)$ in the region D_ε [R, H]. In general there are infinitely many functions with the above properties. However, imposing some additional conditions, the so-called strong asymptotic conditions (SAC) on the rest term $R_N(z)$,

$$f(z)=\sum_{n=0}^{N-1} a_n z^n + R_N(z), \tag{2}$$