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Quantum Group Structure in the Fock Space Resolutions of $\widehat{sl}(n)$ Representations

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Abstract. We describe a complex of Wakimoto-type Fock space modules for the affine Kac–Moody algebra $\widehat{sl}(n)$. The intertwining operators that build the complex are obtained from contour integrals of so-called screening operators. We show that a quantum group structure underlies the algebra of screening operators. This observation greatly facilitates the explicit determination of the intertwiners. We conjecture that the complex provides a resolution of an irreducible highest weight module in terms of Fock spaces.

1. Introduction

There are basically two procedures for constructing the correlation functions of a given conformal field theory on a general Riemann surface. The first consists of solving a set of differential equations arising from the symmetry structure of the theory. This method has proved to be useful in a number of cases, but progress seems to be limited due to the complicated nature of the differential equations involved. The second procedure, that originates in "the old string days," is purely algebraic in origin and involves the explicit computation of the correlation functions by "sewing" fundamental three point functions. The latter procedure, however, seems only feasible for free field theories.

It has been known for some time that many (even non-free) two dimensional conformal field theories admit a free field realization, albeit their Hilbert space is only a subspace of the total Fock space of these free fields. For the minimal models of the Virasoro algebra [BPZ] this so-called Feigin–Fuchs realization ("Coulomb gas") was used elegantly in [DF1, DF2] to compute the correlation functions on a sphere. Generalization to higher genus surfaces, by sewing, requires a procedure for projecting out irreducible representations from this Fock space. It was realized recently that this projection can be achieved by taking alternating sums over an

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