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Index of Subfactors and Statistics of Quantum Fields

II. Correspondences, Braid Group Statistics and Jones Polynomial

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Abstract. The endomorphism semigroup $\mathrm{End}(M)$ of an infinite factor M is endowed with a natural conjugation (modulo inner automorphisms) $\bar{\rho} = \rho^{-1} \cdot \gamma$, where γ is the canonical endomorphism of M into $\rho(M)$. In Quantum Field Theory conjugate endomorphisms are shown to correspond to conjugate superselection sectors in the description of Doplicher, Haag and Roberts. On the other hand one easily sees that conjugate endomorphisms correspond to conjugate correspondences in the setting of A. Connes. In particular we identify the canonical tower associated with the inclusion $\rho(\mathscr{A}(\mathscr{O})) \subset \mathscr{A}(\mathscr{O})$ relative to a sector ρ. As a corollary, making use of our previously established index-statistics correspondence, we completely describe, in low dimensional theories, the statistics of a selfconjugate superselection sector ρ with 3 or less channels, in particular of sectors with statistical dimension $d(\rho) < 2$, by obtaining the braid group representations of V. Jones and Birman, Wenzl and Murakami. The statistics is thus described in these cases by the polynomial invariants for knots and links of Jones and Kauffman. Selfconjugate sectors are subdivided into real and pseudoreal ones and the effect of this distinction on the statistics is analyzed. The FYHLMO polynomial describes arbitrary 2-channels sectors.

1. Introduction

In a previous paper [19, 18] we established a link between the index theory of subfactors [12] and the statistics of a local quantum field [5]: if a superselection sector is represented by a localized endomorphism ρ of the quasi-local C^*-algebra $\mathscr{A} = \cup \mathscr{A}(\mathscr{O})^-$, then

$$\mathrm{Ind}(\rho)^{1/2} = d(\rho).$$

Here $\mathrm{Ind}(\rho)$ is the index of ρ that may be locally defined as the minimal index of $\rho(\mathscr{A}(\mathscr{O}))$ in $\mathscr{A}(\mathscr{O})$ as soon as ρ is localized in $\mathscr{O}$ and $d(\rho)$ is the statistical dimension

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