Commun. Math. Phys. 130, 217–236 (1990)

Communications in Mathematical Physics

Riemann Surfaces, Clifford Algebras and Infinite Dimensional Groups

A. L. Carey[1], M. G. Eastwood[1] and K. C. Hannabuss[2]

[1] Department of Pure Mathematics, University of Adelaide, Adelaide, South Australia
[2] Balliol College, Oxford OX1 3BJ England

Abstract. We introduce a class of Riemann surfaces which possess a fixed point free involution and line bundles over these surfaces with which we can associate an infinite dimensional Clifford algebra. Acting by automorphisms of this algebra is a "gauge" group of meromorphic functions on the Riemann surface. There is a natural Fock representation of the Clifford algebra and an associated projective representation of this group of meromorphic functions in close analogy with the construction of the basic representation of Kac–Moody algebras via a Fock representation of the Fermion algebra. In the genus one case we find a form of vertex operator construction which allows us to prove a version of the Boson–Fermion correspondence. These results are motivated by the analysis of soliton solutions of the Landau–Lifshitz equation and are rather distinct from recent developments in quantum field theory on Riemann surfaces.

Introduction

This note grew out of conversations between the first named author and John Palmer on applications of the results in [CP]. A cursory reading of that paper and the work of Date et al. [DJKM] reveals that the former should provide a rigorous framework for the latter. The main impediment to this program was to understand the intriguing phrase from [D]: "prepare fermions on an elliptic curve." In finding the recipe for this concoction we soon began to tread on ground already familiar from string theory, namely quantum field theory on Riemann surfaces. However it quickly became apparent that there were crucial differences.

Nevertheless in the early stages we benefitted greatly from lectures of D. Quillen and G. Segal given in Oxford in the spring of 1987 on string theory, Riemann surfaces and Bosonisation. While very similar ideas emerge here we find basic differences which can best be summarised by saying that we are dealing with the "real" case whereas the usual work on string theory deals with the complex case. As our account unwinds it will become clear that this involves more than just the usual distinction between Majorana and Dirac Fermions; nevertheless the same