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Grad ϕ Perturbations of Massless Gaussian Fields

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Abstract. We investigate weak perturbations of the continuum massless Gaussian measure by a class of approximately local analytic functionals and use our general results to give a new proof that the pressure of the dilute dipole gas is analytic in the activity.

1. Introduction, Notation, Results

The classical dipole gas at equilibrium is a difficult system. Many foundational results have been obtained [1–5] but for the purposes of this paper the work of Gawedzki-Kupiainen [4, 5] is most relevant. This was the first paper in which the renormalization group was explicitly used on this problem and this paper is an attempt to improve on their methods and results as a step on the way to other problems. For example we expect these methods to be effective in the analysis of self-repelling walk in four dimensions, screening or its absence in quantum statistical Coulomb systems and the φ_4^4 massless lattice quantum field theory.

Our results are designed to provide a framework for the renormalization group in the context of perturbations of the (continuum) massless Gaussian random field. We will use the dipole gas for motivation.

We will first describe the results and proof omitting some technical aspects and then return to give the definitions and state the results carefully.

The Dipole Gas

We consider N dipoles in a periodic box $\Lambda \subset \mathbb{R}^d$, $d \geqq 1$. Each dipole is described by a position coordinate x and a unit polarization vector $\hat{p}$. We unite these degrees of freedom into $\xi \equiv (x, p)$ and let $dQ(\xi) = dx dS(\hat{p})$, where dS is the normalized Lebesgue measure on the surface of the unit ball. The fundamental objects to study are all

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