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Area-Preserving Diffeomorphisms and Higher-Spin Algebras

E. Bergshoeff[1], M. P. Blencowe[2] and K. S. Stelle[2]

[1] Theorey Division, CERN, CH-1211 Geneva 23, Switzerland

[2] The Blackett Laboratory, Imperial College, London SW7, England

Abstract. We show that there exists a one-parameter family of infinite-dimensional algebras that includes the bosonic $d=3$ Fradkin–Vasiliev higher-spin algebra and the non-Euclidean version of the algebra of area-preserving diffeomorphisms of the two-sphere S^2 as two distinct members. The non-Euclidean version of the area preserving algebra corresponds to the algebra of area-preserving diffeomorphisms of the hyperbolic space $S^{1,1}$, and can be rewritten as $\lim_{N\to\infty} su(N,N)$. As an application of our results, we formulate a new $d=2+1$ massless higher-spin field theory as the gauge theory of the area-preserving diffeomorphisms of $S^{1,1}$.

1. Introduction

Infinite-dimensional Lie algebras play an increasingly important role in the development of theoretical physics. One of the best-known examples is the Virasoro algebra, which underlies the physics of two-dimensional conformal field theories. As such, they are important for string theories and for critical phenomena in certain statistical-mechanical models.

Recently, two new types of infinite-dimensional Lie algebras have become relevant. One of them is the algebra of volume-preserving diffeomorphisms of a manifold $\mathscr{M}$, which we denote by $s\,\mathrm{diff}(\mathscr{M})$. This algebra is a subalgebra of the general diffeomorphism algebra of $\mathscr{M}$ and corresponds to the residual symmetry of an extended object in the light-cone gauge. A basic example of such an algebra is $s\,\mathrm{diff}(S^2)$. This algebra occurs in the description of a spherical membrane, which can be viewed as a gauge theory of $s\,\mathrm{diff}(S^2)$. An interesting feature of the algebra $s\,\mathrm{diff}(S^2)$ is that it can be obtained by taking the limit as $N\to\infty$ of the finite-dimensional Lie algebra $su(N)$, i.e., $s\,\mathrm{diff}(S^2)=\lim_{N\to\infty} su(N)$ [1]. Replacing the gauge theory of $s\,\mathrm{diff}(S^2)$ by a gauge theory of $su(N)$ then provides a form of regularization. The original spherical membrane theory is reobtained in the limit $N\to\infty$.