Commun. Math. Phys. 127, 395-424 (1990)

Ocneanu Cell Calculus and Integrable Lattice Models

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Abstract. We show that cell calculus (first introduced by Ocneanu in the context of relative position of factors) is a technique which permits us to connect different integrable models. It generalizes the Vertex-IRF correspondence.

Since the work of Baxter [Ba1], the number of integrable lattice models (ILM) has increased rapidly. Besides, conformal field theory (CFT) [BPZ, Gi1], full of success, has reached a status where the problem of classifying all CFT is worth being investigated [MS]. In a similar way, it is time to put order in the huge amount of solutions of the Yang-Baxter equation (YBE).

Quantum groups [Ji1, Dr] realize part of this: any highest weight representation of the deformation of a simple Lie algebra yields a solution of YBE. Unfortunately all solutions obtained this way are trigonometric functions of the spectral parameter. Except for the 8 Vertex model, which is associated to the Sklyanin algebra [Sk] [an elliptic deformation of SU(2)], one does not know what the algebraic objects associated to elliptic solutions of YBE are. We should add for completeness that there exist solutions of YBE which are related to algebraic curve of genus greater than 2 [Au-Y, Ba2].

In this short introduction we will restrict ourselves to integrable critical models associated to CFT having central charge $c \leq 1$ because one knows lots of things in this case [Pa.1, Gi.2]. Representation of spin $\frac{1}{2}$ of $\mathscr{U}SU(2)_q$ leads to the 6 Vertex model. Using an IRF-Vertex transformation one obtains an SOS model which can be consistently modified to a RSOS model when q is a root of unity [ABF, Pa.2]. These last models are the ABF models based on the Dynkin diagram A_n . This method does not shed any light on the construction of Pasquier models based on other Dynkin diagrams. On the other hand it would be an important step if one could understand more abstractly how ADE models are built.

In [FG] it has been shown that D_n lattice models can be constructed from the A_n lattice models via a discrete symmetry Z_2 . Using a different point of view, we show that on the lattice these models are linked via an object called a cell system