Commun. Math. Phys. 125, 579–595 (1989)

Communications in Mathematical Physics

Thermodynamics for the Zero-Level Set of the Brownian Bridge*

Dimitri Petritis**

Institut de Physique Théorique, Université de Lausanne, CH-1015 Lausanne, Switzerland

Abstract. The random set of instants where the Brownian bridge vanishes is constructed in terms of a random branching process. The Hausdorff measure supported by this set is shown to be equivalent to the partition function of a special class of disordered systems. This similarity is used to show rigorously the existence of a phase transition for this particular class of disordered systems. Moreover, it is shown that at high temperature the specific free energy has the strong self-averaging property and that at low temperature it has no self-averaging property. The unicity at high-temperature and the existence of many limits at low temperature are established almost surely in the disorder.

1. Introduction

Random walks are thoroughly studied in many areas of physics and applied mathematics. A challenging problem that remains open is the construction of the measure for the self-avoiding walk in low dimension (i.e. $d=2$ or 3). A promising method towards that aim is the study of weakly self-avoiding random walks by suppressing walks with many self-intersections [27, 28]. Counting the self-intersections of a walk led to many new concepts; the most useful one seems to be that of local time. Intuitively, if $X_s, s \in [0, \infty[$ is a random process with values in $\mathbf{R}$, its local time on $x \in \mathbf{R}$ can be defined *formally* as

$$L_t(x) = \int_0^t \delta(X_s - x) ds.$$

For Brownian motion, this concept can be given a rigorous meaning [20] and it turned out that it is intimately connected to the Hausdorff measure of the x-level sets of the Wiener process, i.e. of the random sets of instants where the Brownian motion attains the value x [15]. The study of the weakly self-avoiding walks and their intersections was the starting point of this work [17].

---

* Work supported by the Swiss National Science Foundation

** Permanent address: Institut mathématique, Université de Rennes I, Campus de Beaulieu, F-35042 Rennes Cedex, France