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On a Certain Value of the Kauffman Polynomial*

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Abstract. If $F_L(a,x)$ is the Kauffman polynomial of a link L we show that $F_L(1, 2\cos 2\pi/5)$ is determind up to a sign by the rank of the homology of the 2-fold cover of the complement of L. This value corresponds to a certain Wenzl subfactor defined by the Birman-Wenzl algebra, which we describe in simple terms. There also corresponds a "solvable" model in statistical mechanics similar to the 5-state Potts model. It is the 5-state case of a general model of Fateev and Zamolodchikov.

Introduction

This paper is intended to demonstrate the fruitfulness of a correspondence which is now emerging between knot theory, von Neumann algebras, and statistical mechanics. We begin by describing a simpler example of this correspondence which is precisely generalized in this paper. It was already largely present in [J1].

If $V_{\mathbf{L}}(t)$ is the polynomial of [J1] for a tame oriented link $\mathbf{L}$ in $\mathbb{R}^3$ then $V_{\mathbf{L}}$ can be calculated as the (normalized) trace of a braid α whose closure $\hat{\alpha}$ is $\mathbf{L}$, in representations of the braid groups that arose in von Neumann algebras. In order to construct interesting subfactors of II_1 factors the author in [J2] used what was essentially the following device: Find a suitable Hilbert space representation π of the infinite braid group $B_\infty = \langle \sigma_1, \sigma_2, \dots;\ \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1},\ \sigma_i\sigma_j = \sigma_j\sigma_i$ for $|i-j| \geqq 2\rangle$ such that $\pi(B_\infty)$ generates a type II_1 factor. The subgroup of B_∞ generated by $\sigma_2, \sigma_3, \dots$ should then generate a subfactor whose index can, under the right circumstances, be calculated. It was some representations that made the subfactor construction work which were used to construct the link invariant $V_{\mathbf{L}}(t)$.

In the case where the subfactor had *integer index* (for the definition of index, see [J2], it was possible to use a braid group representation that had already been (essentially) discovered by Temperley and Lieb in [TL] in their proof of the equivalence of the Potts and ice-type models in 2-dimensional square lattice

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