Commun. Math. Phys. 125, 439–457 (1989)

Communications in
Mathematical
Physics

Dispersing Billiards Without Focal Points on Surfaces are Ergodic

A. Krámli[1], N. Simányi[2,*], and D. Szász[2,*]

[1] Computer and Automation Institute, Hungarian Academy of Sciences, Budapest
[2] Mathematical Institute, HAS, H-1364 Budapest, POB 127, Hungary

Abstract. Billiards are considered on two-dimensional, smooth, compact Riemannian manifolds with dispersing scatterers. We prove that these billiards are ergodic if only Vetier's conditions for the absence of focal points hold.

0. Introduction

Consider a two-dimensional twice continuously differentiable, compact, closed oriented Riemannian manifold. A simply connected, open subset of the surface will be called a scatterer if its boundary is a twice continuously differentiable curve with strictly positive geodesic curvature from inside. If we have a number of disjoint scatterers, then we call the complement Q of their union a billiard table.

The billiard on Q is a dynamical system corresponding to a motion with unit velocity along geodesics inside Q combined with elastic reflection at the scatterers, i.e. on ∂Q. In particular, if the surface is the torus, then we recover the celebrated Sinai billiard.

In billiards on a surface with scatterers two kinds of behavior can compete: the sufficiently good mixing one caused by the scatterers and a possibly integrable one inside the surface. In 1982 Vetier (Vi (1982), $i=1,2$) was able to give conditions under which no focal points arise and thus mixing prevails. He also established the hyperbolic theory for these billiards. Under his conditions the Lyapunov exponents are uniformly bounded away from zero implying the a.e. existence of fibers and properties called the absolute continuity of the foliations. His main conclusion is that the ergodic components of these billiards are positive V(1987).

Here we prove that under Vetier's conditions the billiard is ergodic. By the traditional Hopf-Sinai strategy this follows from a version of the fundamental theorem and, in fact, this is the main result (Theorem 5.1) of our paper.

The proof of the fundamental theorem we separate into two parts. The chief aim of the first, geometric part is to formulate lemmas permitting us to think and

* Partially supported by the Hungarian National Foundation for Scientific Research, grant No. 819/1 and by Central Research Fund, grant No. 501/5/4/1984