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The Topological Sigma Model

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Abstract. We obtain the invariants of Witten's topological σ-model by gauge fixing a topological action and using BRST symmetry. The fields and the BRST formalism are interpreted geometrically.

1. Introduction

In [1], Witten introduced the idea of topological quantum field theory and showed how to obtain the Donaldson polynomials from his action on four dimensional vector potentials. In [2], we showed how to obtain his result by starting with a purely topological action $I_{\text{top}} = (8\pi)^{-2} \int_{M_4} \operatorname{Tr} F \wedge F$, i.e. the second chern class. Since the action is a constant function on vector potentials, the path integral $\int \mathscr{D}A \exp -I_{\text{top}}$ needs interpretation. We did so by choosing appropriate gauge functions and applying the BRST formalism. The topological invariance followed from the BRST symmetry. This symmetry and the ghost fields introduced by gauge fixing has a geometric interpretation on $M_4 x \mathscr{A}/\mathscr{G}$, where M_4 is the 4-manifold and $\mathscr{A}/\mathscr{G}$ is the orbit space of vector potentials equivalent under gauge transformations.

In this paper we present the case of Witten's topological σ-model [3] in the same spirit. Again we start with a topological action, which we gauge fix. The resultant ghost fields have a geometric interpretation and the cocycles of the BRST symmetry lead to topological invariants.

As we might expect, the moduli space of (anti) selfdual Yang Mills fields is replaced by the space of (antiholomorphic) pseudo-holomorphic maps (in the sense of Gromov [4]) and the topological invariants are multilinear maps on the cohomology of M.

As in the case of the topological Yang-Mills theory, the topological properties of "physical observables" computed by functional integration are guaranteed from the Ward identities of the BRST symmetry corresponding to the enlarged gauge

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