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Analysis of Low-Temperature Phase Diagram of the Microemulsion Model

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Dedicated to Roland Dobrushin

Abstract. The low-temperature phase diagram of the microemulsion model is constructed for such values of its parameters when the model has only layered ground states.

1. Introduction

The microemulsion model is a lattice 3-dimensional spin model of statistical mechanics. It was proposed by Widom in [1] for the description of the microfilm structure in a mixture "oil-water." Also he showed that for some values of the mixture parameters the model can be reduced to a lattice spin model given by the Hamiltonian:

$$H(\varphi(\mathbb{Z}^3)) = -I \sum_{\mathrm{dist}(x,y)=1} \varphi(x)\varphi(y) - J \sum_{\mathrm{dist}(x,y)=\sqrt{2}} \varphi(x)\varphi(y) - K \sum_{\mathrm{dist}(x,y)=2} \varphi(x)\varphi(y), \tag{1.1}$$

where $x, y \in \mathbb{Z}^3$, $\varphi(\mathbb{Z}^3)$ is a configuration on $\mathbb{Z}^3$ taking values in the set $\{\pm 1\}$, $J = 2K$. We shall consider a more general model with any I, J, K. Clearly (see [2]) it is sufficient to consider only positive values of I.

First we investigate the ground states of the Hamiltonian (1.1). With this purpose we put

$$O(x) = \{y \in \mathbb{Z}^3 : \mathrm{dist}(x, y) \leqq 1\}$$

and

$$U(\varphi(O(o))) = -0.5I \sum_{\substack{\mathrm{dist}(x,y)=1 \\ x,y \in O(o)}} \varphi(x)\varphi(y) - 0.5J \sum_{\substack{\mathrm{dist}(x,y)=\sqrt{2} \\ x,y \in O(o)}} \varphi(x)\varphi(y) - K \sum_{\substack{\mathrm{dist}(x,y)=2 \\ x,y \in O(o)}} \varphi(x)\varphi(y). \tag{1.2}$$