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An Estimate from Above of the Number of Periodic Orbits for Semi-Dispersed Billiards

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Abstract. For a large class of semi-dispersed billiards an exponential estimate from above is found for the number of periodic points of the billiard ball map.

1. Introduction and Main Results

Let Q be a domain (bounded or unbounded) in $\mathbb{R}^d$, $d \geqq 2$, with the boundary

$$\partial Q = \Gamma = \Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_s \quad (s \geqq 3),$$

where each Γ_i is a compact convex C^2-smooth $(d-1)$-dimensional submanifold of $\mathbb{R}^d$ with piecewise smooth boundary $\partial \Gamma_i$, and

$$\Gamma_i \cap \Gamma_j \subset \partial \Gamma_i \cup \partial \Gamma_j$$

whenever $i \neq j$. Each $\partial \Gamma_i$ is the union of a finite number of compact $(d-2)$-dimensional submanifolds of $\mathbb{R}^d$. If $\partial \Gamma_i \neq \emptyset$, then clearly Γ_i is the boundary of a compact convex domain in $\mathbb{R}^d$.

Main Assumption. In the sequel we assume that each Γ_i is contained in the boundary of a convex domain in $\mathbb{R}^d$. Therefore if K_i is the convex hull of Γ_i, then $\Gamma_i \subset \partial K_i$.

The points of

$$\mathring{\Gamma} = (\Gamma_1 \backslash \partial \Gamma_1) \cup \cdots \cup (\Gamma_s \backslash \partial \Gamma_s)$$

will be called *regular points* of Γ. For $q \in \mathring{\Gamma}$ we denote by $N(q)$ the *normal unit* vector to Γ at q directed to the interior of Q. With respect to this framing the second fundamental form of Γ is non-negative definite at each $q \in \mathring{\Gamma}$.

We consider the billiard in Q, that is the dynamical system generated by the motion of material point in Q (see [4, 13]). The point is moving with constant velocity in the interior of Q with reflections at ∂Q according to the rule "the angle of incidence is equal to the angle of reflection."

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