Commun. Math. Phys. 124, 153–168 (1989)

Communications in
Mathematical
Physics

A Renormalization Group Proof of Perturbative Renormalizability

T. R. Hurd*

Department of Mathematics, University of British Columbia, Vancouver, Canada, V6T 1Y4

Abstract. This paper presents a proof of bounds on the renormalized perturbation expansion of the euclidean $\lambda\phi_4^4$ theory. Its aim is partly pedagogical: by combining the insights and techniques of numerous authors it is now possible to define the perturbation expansion and bound it in a very few pages. The present version is based on the renormalized tree expansion adapted to the continuous renormalization group: all detailed results are proved by induction on the size of the tree. The continuous RG version presented here has one big advantage over the discrete RG version discussed elsewhere. In the continuous version, a tree has a more restrictive structure: there is a one-to-one correspondence between forks of the tree and lines of Feynman graphs. This extra structure eliminates the need to introduce Feynman graphs in the first place. It also reduces the number of cases to be analyzed at a given inductive step and simplifies the combinatorical estimates.

1. Introduction

It is recognized by now that renormalization is best understood in the framework of Wilson's renormalization group [12, 13]. This is well exhibited in the setting of constructive quantum field theory by the resent work Gawedzki and Kupiainen and Feldman et al. on the Gross-Neveu$_2$ and infra-red ϕ_4^4 models [8, 6]. Another important and perhaps simpler realization of RG ideas has been in renormalized perturbation theory.

The traditional approach to renormalization theory, highlighted in such landmark papers as [3, 1, 11, 9, 14, 2], has been based on Feynman graphs and the idea that infinities can be cancelled by the introduction of infinite counterterms into the lagrangian. The most refined formulation of the renormalized Feynman graph expansion was the Zimmermann forest formula, which defines the renormalization

* Research supported by the Natural Sciences and Engineering Research Council
Present adress: Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S 4K1