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Determinants of Dirac Operators and Thirring Model Partition Functions on Riemann Surfaces with Boundaries

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Abstract. The partition function of the Thirring model on a Riemann surface with boundaries is calculated using the method of Freedman and Pilch by introducing an auxiliary vector potential in the path integral of fermion representation. The Hodge decomposition on manifolds with boundaries is used to integrate over the harmonic forms. The result agrees with the bosonized calculation. The determinants of Dirac operators with mixed Neveu-Schwarz and Ramond boundary conditions are expressed in terms of the Riemann ϑ-functions of the doubled surface.

1. Introduction

The determinants of the Laplacians and the Dirac operators on a closed Riemann surface have been studied extensively in the Polyakov formulation of the closed string theory [1, 2]. For the open strings, the world sheets are Riemann surfaces with boundaries [3]. Determinants of the Laplacians with both Dirichlet and Neumann boundary conditions were calculated and used in the study of open bosonic strings [4, 5]. In this paper, we calculate the determinants of Dirac operators with Neveu-Schwarz and Ramond boundary conditions. These determinants are useful in the open superstring theory, where the bosonic coordinates of the world sheet live on a Riemann surface with boundaries.

The determinants of Dirac operators coupled to an abelian gauge field on a closed Riemann surface with boundaries were calculated by Quillen's method of holomorphic anomaly [6, 2]. The result was soon used to prove the bosonization of first order fermionic systems on a closed Riemann surface of higher genus [7, 8]. Recently it was also used in the calculation of the partition function of the massless Thirring model in the fermion representation by introducing an auxiliary vector potential [9]. The same method works for the Thirring model on a Riemann surface with boundaries. We will calculate its partition using the fermion determinant and show that it agrees with the bosonized calculation.