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Quantum BRST Charge for Quadratically Nonlinear Lie Algebras

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Abstract. We consider the construction of a nilpotent BRST charge for extensions of the Virasoro algebra of the form $\{T_a, T_b\} = f_{ab}{}^c T_c + V^{cd}_{ab} T_c T_d$, (classical algebras in terms of Poisson brackets) and $[T_a, T_b] = h_{ab} I + f_{ab}{}^c T_c + V^{cd}_{ab}(T_c T_d)$ (quantum algebras in terms of commutator brackets; normal ordering of the product $(T_c T_d)$ is understood). In both cases we assume that the set of generators $\{T_a\}$ splits into a set $\{H_i\}$ generating an ordinary Lie algebra and remaining generators $\{S_\alpha\}$, such that only the $V^{ij}_{\alpha\beta}$ are nonvanishing. In the classical case a nilpotent BRST charge can always be constructed; for the quantum case we derive a condition which is necessary and sufficient for the existence of a nilpotent BRST charge. Non-trivial examples are the spin-3 algebra with central charge $c = 100$ and the $so(N)$-extended superconformal algebras with level $S = -2(N-3)$.

1. Introduction

Over the past few years it has become clear that conformal field theories in two dimensions play an important role in string theories and in statistical systems at the critical point (a large number of relevant papers can be found in the reprint volume [1]). Each conformal field theory is built from a set of representations of the two-dimensional conformal algebra, which is the product of two copies of the Virasoro algebra. However, in actual models there is often more symmetry than just conformal invariance. In fact, all rational conformal field theories correspond to the Virasoro algebra or some extension of it ([2, 3]). In general such extended algebras are generated by a finite set of currents of definite conformal dimension. A systematic study of finitely generated conformal algebras was initiated by Zamolodchikov in ref. [4] and has been developed further by many authors.

The extended algebras that turn up in $d = 2$ conformal field theory are quantum mechanical, i.e. they describe the (anti)commutation relations of operator-valued fields. The classical versions of these algebras, where the bracket is interpreted as a Poisson or Dirac bracket, are relevant in the study of certain hierarchies of completely integrable systems generalizing the KdV-hierarchy [5, 6].