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Algebraic Study on the Super-KP Hierarchy and the Ortho-Symplectic Super-KP Hierarchy

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Abstract. Bilinear residue formulas are established for the super-KP hierarchy and the ortho-symplectic super-KP hierarchy. Furthermore, superframes corresponding to the ortho-symplectic super-KP hierarchy are completely characterized. Soliton solutions to the super-KP hierarchy are given.

1. Introduction

This paper is devoted to algebraic study of super-wave functions and soliton solutions of the super Kadomtsev–Petviashvili (SKP) hierarchy and the ortho-symplectic (OSp) SKP hierarchy.

The SKP hierarchy was first introduced by Manin–Rudal [12] and was extensively studied by Ueno–Yamada [17–20], Yamada [21], Mulase [13], Ikeda [9] and Radul [14]. Especially, in [19] we proved that the SKP hierarchy equivalently leads to the super-Grassmann equation that connects a point in the universal super-Grassmann manifold *USGM* with an initial data of a solution. In that argument, the Birkhoff (Riemann–Hilbert) decomposition in the group of super-microdifferential operators plays a key role. However this operator formalism is rather inconvenient for treating geometrical solutions such as soliton solutions and super-quasi-periodic solutions. We therefore require a super-wave function, as in the case of the ordinary soliton theory.

The theory of the KP hierarchy itself is explained as follows [2, 6, 15, 16]: Let $\mathscr{R}$ be the ring of formal power series over $\mathbf{C}$, $\mathscr{R}=\mathbf{C}[[x,t]]$ (x is a space variable and $t=(t_1,t_2,t_3,\dots)$ an infinite number of time variables.). The algebra $\mathscr{R}$ is a differential algebra with a derivation $\partial_x=\partial/\partial x$. By $\mathscr{E}_{\mathscr{R}}$ we denote the ring of microdifferential operators over $\mathscr{R}$,

$$\mathscr{E}_{\mathscr{R}}=\mathscr{R}((\partial_x{}^{-1}))=\left\{\sum_{-\infty<\nu\ll+\infty} p_\nu(x,t)\partial_x{}^{\nu}\,|\,p_\nu(x,t)\in\mathscr{R}\right\}.$$

A wave operator

$$W=W(x,t,\partial_x)=\sum_{j=0}^{\infty} w_j(x,t)\partial_x{}^{-j}\quad (w_0=1) \tag{1.1}$$