Commun. Math. Phys. 124, 23–55 (1989)

Communications in Mathematical Physics



# Homogeneous Kähler Manifolds and $T$-Algebras in $N = 2$ Supergravity and Superstrings

S. Cecotti

CERN, CH-1211 Geneva 23, Switzerland and International School for Advanced Studies, Trieste, Italy

**Abstract.** Motivated by the problem of the moduli space of superconformal theories, we classify all the (normal) homogeneous Kähler spaces which are allowed in the coupling of vector multiplets to $N = 2$ SUGRA. Such homogeneous spaces are in one-to-one correspondence with the homogeneous quaternionic spaces ($\neq \mathbb{H}H^n$) found by Alekseevskii. There are two infinite families of homogeneous non-symmetric spaces, each labelled by two integers. We construct explicitly the corresponding supergravity models. They are described by a *cubic* function $F$, as in flat-potential models. They are Kähler–Einstein if and only if they are symmetric. We describe in detail the geometry of the relevant manifolds. They are Siegel (bounded) domains of the first type. We discuss the physical relevance of this class of bounded domains for string theory and the moduli geometry. Finally, we introduce the $T$-algebraic formalism of Vinberg to describe in an efficient way the geometry of these manifolds. The homogeneous spaces allowed in $N = 2$ SUGRA are associated to rank 3 $T$-algebras in exactly the same way as the symmetric spaces are related to Jordan algebras. We characterize the $T$-algebras allowed in $N = 2$ supergravity. They are those for which the *ungraded* determinant is a polynomial in the matrix entries. The Kähler potential is simply minus the logarithm of this "naive" determinant.

## 1. Introduction

One promising approach [1] to the geometry of the moduli space for an abstract $2d$ superconformal field theory is the study of the low-energy supergravity corresponding to the superstring model defined by this theory. Probably, the most interesting case is that of (2, 2) superconformal systems, which according to a well-motivated conjecture by Gepner [2] should correspond to a $\sigma$-model on a Calabi–Yau manifold. Many results on the moduli spaces for (2, 2) systems were obtained using this method in refs. [3, 4]. Indeed, it turns out that many problems in the moduli theory were already worked out in the context of supergravity, and hence many