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Higher Spin Fields and the Gelfand–Dickey Algebra

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Abstract. We show that in 2-dimensional field theory, higher spin algebras are contained in the algebra of formal pseudodifferential operators introduced by Gelfand and Dickey to describe integrable nonlinear differential equations in Lax form. The spin 2 and 3 algebras are discussed in detail and the generalization to all higher spins is outlined. This provides a conformal field theory approach to the representation theory of Gelfand–Dickey algebras.

1. Introduction

Recently Zamolodchikov investigated additional symmetries in 2-dimensional conformal field theory generated by higher spin local currents [1]. It is known that in two dimensions the independent components of the stress energy tensor $T(z)$, $\bar{T}(\bar{z})$, generate the (infinite) algebra of conformal transformations. The operator product expansion for the fields $T(z)$ has the form

$$T(z)T(w) = \frac{c}{2(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots, \tag{1}$$

where $\cdots$ denote all nonsingular terms. Introducing Fourier components $L_n (n \in \mathbb{Z})$, we obtain the Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{2.3!}(n^3 - n)\delta_{n+m,0}. \tag{2}$$

Primary conformal fields $\varphi_\Delta(z)$ with conformal weight Δ are characterized by the operator product expansion

$$T(z)\varphi_\Delta(w) = \frac{\Delta}{(z-w)^2}\varphi_\Delta(w) + \frac{\partial \phi_\Delta(w)}{z-w} + \cdots. \tag{3}$$

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