Commun. Math. Phys. 123, 617–625 (1989)

Communications in Mathematical Physics

On the Mickelsson–Faddeev Extension and Unitary Representations

Doug Pickrell
Mathematics Department, University of Arizona, Tucson, AZ 85721, USA

Abstract. The Mickelsson–Faddeev extension is a 3-space analogue of a Kac-Moody group, where the central charge is replaced by a space of functions of the gauge potential. This extension is a pullback of a universal extension, where the gauge potentials are replaced by operators in a Schatten ideal, as in non-commutative differential geometry. Our main result is that the universal extension cannot be faithfully represented by unitary operators on a separable Hilbert space. We also examine potential consequences of the existence of unitary representations for the Mickelsson–Faddeev extension.

Section 1. Introduction

The Mickelsson–Faddeev extension, denoted by $\hat{M}$ in this paper, is a certain distinguished noncentral abelian extension of the Hamiltonian gauge (or equal time current) group $C^{\infty}(X, G)$:

$$0 \to F \to \hat{M} \to C^{\infty}(X, G) \to 0.$$

(see [Mi2 or Fr] and the references cited there).

The kernel of the extension, F, consists of a certain class of functions of a gauge potential, the class depending on the dimension of X, and it arises in the process of regularizing the gauge (or current) operators. An intriguing question is whether $\hat{M}$ can be represented by unitary operators on a Hilbert space. When X is one dimensional the answer is yes, for then $\hat{M}$ is essentially the Kac-Moody extension and regularization amounts to normal ordering. In higher dimensions regularization involves a multiplicative renormalization, and it is not clear whether this is compatible with unitarity (it is possible to construct nonunitary representations—see [Se or MR]).

One objective of this paper is to cast the Mickelsson–Faddeev extension in a form which is amenable to analysis, at least for X of dimension three. In this case we can take F to consist of real valued affine functions modulo a copy of the integers. We can then think of the extension as a two stage process, the first analytical, the second topological. The first stage is a *topologically* trivial extension

$$0 \to V \to M \to C^{\infty}(X, G) \to 0,$$