Commun. Math. Phys. 122, 411–438 (1989)

Communications in Mathematical Physics

Perturbation of Embedded Eigenvalues in the Generalized N-Body Problem

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Abstract. We discuss the perturbation of continuum eigenvalues without analyticity assumptions. Among our results, we show that generally a small perturbation removes these eigenvalues in accordance with Fermi's Golden Rule. Thus, generically (in a Baire category sense), the Schrödinger operator has no embedded non-threshold eigenvalues.

I. Introduction

It is well known [R–S1] that a one-body Schrödinger operator $-\Delta + V(x)$, where V is sufficiently well behaved at infinity, cannot have eigenvalues λ embedded in the continuous spectrum (except possibly at threshold, $\lambda = 0$). The situation is quite different in the N-body problem where continuum eigenvalues not only can exist, but do indeed exist in important physical situations: The operator $H_0 = -\Delta_1 - \Delta_2 - 2/|x_1| - 2/|x_2|$ in $L^2(\mathbb{R}^6)$ (describing the Helium atom without electronic repulsion) has eigenvalues embedded in the continuous spectrum. While this example has an obvious symmetry, such symmetry is not necessary for the existence of embedded eigenvalues. An example in [F–H–HO–HO] can be modified to produce an embedded eigenvalue where no symmetry is apparent.

In [How1, 2] and [S1], analyticity assumptions are made which allow the treatment of embedded eigenvalues using the perturbation theory developed for use with isolated eigenvalues. The major idea in this theory is that when a small perturbation βW is added to the Schrödinger operator H, the continuum eigenvalue E_0 turns into a "resonance," $E_0(\beta)$, which, while not necessarily an eigenvalue of $H + \beta W$, is a pole in the analytic continuation of certain matrix elements $(\varphi, (H + \beta W - z)^{-1}\varphi)$ of the resolvent. The function $E_0(\beta)$ is analytic in β for $|\beta|$ small. $E_0(\beta)$ has an imaginary part which appears first to second order in β:

$$\left| \operatorname{Im} \frac{d^2 E_0(\beta)}{d\beta^2} \right|_{\beta=0} = 2\pi \frac{d}{dE} (W\psi_0, P(E) W\psi_0)|_{E=E_0}, \tag{1.1}$$

* Supported in part by NSF Grant DMS 8602826