Commun. Math. Phys. 122, 393–410 (1989)

Communications in
Mathematical
Physics

The Exceptional Jordan Algebra and the Superstring

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Abstract. Two representations of the exceptional Jordan algebra are presented, one in terms of bose vertex operators, and the other in terms of superstring vertex operators in bosonised form, including their BRST ghost contributions. It is also shown how the non-exceptional Jordan algebras may be constructed similarly.

1. Introduction

Over the years it has been suggested that Jordan algebras, which originated in the search for alternative formulations of quantum mechanics [1], might play a rôle in the theory of colour and confinement [2], in supersymmetry [3], and more recently, in the theory of superstrings [4–11]. In particular, physicists have been fascinated by the existence of the unique, exceptional Jordan algebra which may be represented in terms of three dimensional hermitian matrices with octonion elements [12]. In fact Jordan algebras have led to insights more of a mathematical than physical nature [13]. Perhaps string theory will be different, since it already has strong connections with algebras in several ways. In particular, the vertex operators of string theory are able, on the one hand, to represent the couplings of strings, and on the other, to provide certain representations of Lie and Kac-Moody algebras [14–16], and other interesting algebras such as the Virasoro algebra and the algebra associated with the Fischer–Griess monster group [17]. It is therefore natural to ask if there is a connection between vertex operators and Jordan algebras. Indeed, there is a suspicion that there should be a relationship between the exceptional Jordan algebra and the vertex operators of the superstring [5]. Our aim in this article is to make this connection more concrete.

Since our main motivation is to uncover a relationship between the superstring vertices [18–22] and the exceptional Jordan algebra, we shall use this particular example to illustrate our ideas. However, the ideas are certainly extendable to the other Jordan algebras and the relevant details will be explained in Sect. (4).

Abstractly, a Jordan algebra [12] is a real (finite dimensional) vector space J together with a symmetric, but not necessarily associative, vector product satisfying