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Large Field Renormalization. II. Localization, Exponentiation, and Bounds for the R Operation

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Abstract. In this paper we conclude the discussion of the basic properties and bounds for the **R** operation. This allows us to complete the proof of the ultraviolet stability of four-dimensional pure gauge field theories, as formulated in Theorem 1.

Introduction

In this paper we perform all the remaining operations defining **R**, like cancellations of the large terms in numerators and denominators in (1.100) [IV], localizations in disjoint large field regions, and exponentiations. Then we prove bounds for the obtained expressions. The main bounds are combinatorial bounds proving convergence of the whole expansion defined by the **R** operation. This completes the proof of the inductive representation of the effective densities, the representation described in detail in Sect. 2 [III]. Thus we prove the following theorem:

Theorem 1. *If the sequence of the effective coupling constants is contained in an interval* $]0, \gamma]$ *with a sufficiently small positive* γ*, then the effective densities* ρ_k *have the form, and satisfy all the conditions and bounds, described in Sect. 2 [III].*

This is the main result of the whole sequence of papers of the present author on non-Abelian gauge field theories. Theorem 2 of [I] allows us to remove the assumption on the effective coupling constants in the above theorem, because this assumption follows from the basic inequality (0.31) [II], which is the result of Theorem 2. The proof of Theorem 2, which is based on second order perturbative calculations, is very awkward and long in the context of the renormalization group approach to lattice gauge field theories, and has not been published yet, so we have Theorem 1 with the assumption. As an immediate consequence of this theorem, we get the ultraviolet stability bounds of the same type as for superrenormalizable models in [16]:

$$\chi_k \exp\left[-\frac{1}{g_k^2} A(U_k) - E_- |T_\eta| \right] \leqq \rho_k \leqq \exp E_+ |T_\eta|, \tag{0.1}$$

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