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The Group of Local Biholomorphisms of $\mathbb{C}^1$ and Conformal Field Theory in the Operator Formalism

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Abstract. Motivated by the operator formulation of conformal field theory on Riemann surfaces, we study the properties of the infinite dimensional group of local biholomorphic transformations (conformal reparametrizations) of $\mathbb{C}^1$ and develop elements of its representation theory.

1. Introduction

String theory [1] has provided and continues to provide motivation for much interesting mathematics. It suffices to recall the vertex operator construction of representations of Kac-Moody algebras [2], the impulse it gave to the study of the global geometry of the moduli space of Riemann surfaces, and the surge of interest in two dimensional conformal field theory (CFT), inspired mainly by the role of CFT in string theory. Indeed, recent developments, to mention only [3, 4], have raised CFT to the role of the main technical tool in the study of quantum string dynamics, at least within the first quantized approach. It is also worth adding that CFT has considerable interest in itself, for it has a quite intricate mathematical structure, which is nonetheless much more tractable than that of more general quantum field theories. CFT has moreover found important applications within the theory of two dimensional critical phenomena.

The present work was motivated mainly by recent developments concerning the operator formulation of CFT on higher genus Riemann surfaces [5, 6]. One of the characteristics of these approaches is the use they make of a space $\mathscr{M}$ of geometrical data, consisting in the simplest case of triples (M, P, z), with M a (closed) Riemann surface, P a distinguished point on M, and z a local uniformizer at P, i.e. a holomorphic function in a neighborhood of P with $dz(P) \neq 0$. The freedom of choosing any z with the above properties corresponds to the action on $\mathscr{M}$ of an infinite dimensional complex continuous group, which we refer to in the following as the group $\mathscr{G}$ of local biholomorphisms; it would seem natural to view $\mathscr{M}$ as a principal fiber bundle with structure group $\mathscr{G}$. Moreover, considering a

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